

Future Calculus Student,

Next year, you will be involved in the most academically challenging class of your high school career. Why is Calculus so challenging? It is challenging because it requires you to use all of your academic and problem solving skills; reading, listening, writing, and mathematical skills. Most of you will not find the arithmetic skills to be that difficult. What will be challenging is learning the vocabulary and actually putting in the daily and/or weekly effort it takes to be successful. You should be prepared to spend upwards of 3-4 hours weekly working on Calculus outside of the classroom.

To prepare you for success, I've created this summer assignment to help you to be mathematically prepared for the class. The topics inside are intended to be a review of the essential skills necessary to make your year in Calculus a success. I'm not interested in wasting your time and I hope you aren't interested in wasting mine either. I assure you I'll be working hard to make passing the AP Calculus Exam as easy as possible. Regardless, your effort will be the most important ingredient.

As far as actually preparing yourself ahead of time, I also recommend picking up a copy of Barron's AP Calculus. It is a review and study guide for preparing for the AP Calculus Exam and I find the book to be the best of the possible practice guide books. It is broken down by topics in the same format and timeline that I'll be teaching the course. YOU DO NOT need to purchase the book as it is only a recommendation. You also do not need to buy the newest edition as they haven't changed in many, many years. It is possible to buy older copies for just a few dollars. If you buy it, read through it. Try to teach yourself about the first topic of Limits. Use YouTube videos as helpful tutoring guides as well.

Good luck next year! I expect to receive this summer assignment completed on the first day of school! Let's earn some college credit and save yourself or your family 1000s of dollars in college tuition!

Email me if you have questions or concerns at dkohler6245@columbus.k12.oh.us and I'll respond as soon as possible.

Sincerely,
Mr. Doug Kohler

AP Calculus AB
2017 Summer Review Packet Assignment
Whetstone High School

Due: On the first day of school!

Name _____

Directions/Rubric:

- 1) There are 14 sections. Each one will be counted as a homework grade. That's 14 different homework assignments in one packet!
- 2) Your work must be shown on a separate piece of paper or in the packet if it is done neatly.
- 3) It is due on the first day of school. NO EXCEPTIONS!
- 4) Contact me via email if you have questions. Dkohler6245@columbus.k12.oh.us
- 5) Do not wait until the last minute.

Assignment 1: Functions

FUNCTIONS

To evaluate a function for a given value, simply plug the value into the function for x .

Recall: $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read “ f of g of x ” Means to plug the inside function (in this case $g(x)$) in for x in the outside function (in this case, $f(x)$).

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find $f(g(x))$.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each.

1. $f(2) =$ _____ 2. $g(-3) =$ _____ 3. $f(t+1) =$ _____

4. $f[g(-2)] =$ _____ 5. $g[f(m+2)] =$ _____ 6. $[f(x)]^2 - 2g(x) =$ _____

Let $f(x) = \sin(2x)$ Find each exactly.

7. $f\left(\frac{\pi}{4}\right) =$ _____ 8. $f\left(\frac{2\pi}{3}\right) =$ _____

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each.

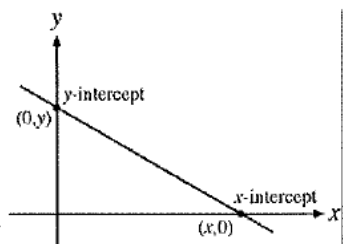
9. $h[f(-2)] =$ _____ 10. $f[g(x-1)] =$ _____ 11. $g[h(x^3)] =$ _____

Assignment 2: Intercepts of Graphs

INTERCEPTS OF A GRAPH

To find the x-intercepts, let $y = 0$ in your equation and solve.

To find the y-intercepts, let $x = 0$ in your equation and solve.



Example: Given the function $y = x^2 - 2x - 3$, find all intercepts.

x-int. (Let $y = 0$)

$$0 = x^2 - 2x - 3$$

$$0 = (x - 3)(x + 1)$$

$$x = -1 \text{ or } x = 3$$

x - intercepts $(-1, 0)$ and $(3, 0)$

y-int. (Let $x = 0$)

$$y = 0^2 - 2(0) - 3$$

$$y = -3$$

y - intercept $(0, -3)$

Find the x and y intercepts for each.

12. $y = 2x - 5$

13. $y = x^2 + x - 2$

14. $y = x\sqrt{16 - x^2}$

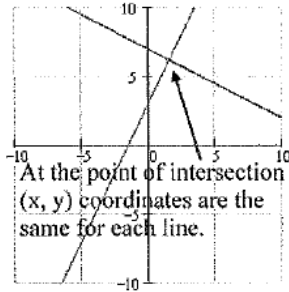
15. $y^2 = x^3 - 4x$

Assignment 3: Points of Intersection

POINTS OF INTERSECTION

Use substitution or elimination method to solve the system of equations.

Remember: You are finding a POINT OF INTERSECTION so your answer is an ordered pair.



CALCULATOR TIP

Remember you can use your calculator to verify your answers below. Graph the two lines then go to CALC (2nd Trace) and hit INTERSECT.

Example: Find all points of intersection of $x^2 - y = 3$
 $x - y = 1$

ELIMINATION METHOD

Subtract to eliminate y

$$x^2 - x = 2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2 \text{ or } x = -1$$

Plug in $x=2$ and $x=-1$ to find y

Points of Intersection: $(2, 1)$ and $(-1, -2)$

SUBSTITUTION METHOD

Solve one equation for one variable.

$$y = x^2 - 3$$

$$y = x - 1$$

Therefore by substitution $x^2 - 3 = x - 1$

$$x^2 - x - 2 = 0$$

From here it is the same as the other example

Find the point(s) of intersection of the graphs for the given equations.

16. $x + y = 8$
 $4x - y = 7$

17. $x^2 + y = 6$
 $x + y = 4$

18. $x = 3 - y^2$
 $y = x - 1$

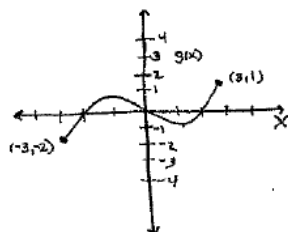
Assignment 4: Domain & Range

DOMAIN AND RANGE

Domain – All x values for which a function is defined (input values)

Range – Possible y or Output values

EXAMPLE 1



a) Find Domain & Range of $g(x)$.

The domain is the set of inputs for the function. Input values run along the horizontal axis.

The furthest left input value associated with a pt. on the graph is -3. The furthest right input values associated with a pt. on the graph is 3.

So Domain is $[-3, 3]$, that is all reals from -3 to 3.

The range represents the set of output values for the function. Output values run along the vertical axis. The lowest output value of the function is -2. The highest is 1. So the range is $[-2, 1]$, all reals from -2 to 1.

EXAMPLE 2

Find the domain and range of $f(x) = \sqrt{4-x^2}$
Write answers in interval notation.

DOMAIN

For $f(x)$ to be defined $4-x^2 \geq 0$.

This is true when $-2 \leq x \leq 2$

Domain: $[-2, 2]$

RANGE

The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0.

Range: $[0, \infty)$

Find the domain and range of each function. Write your answer in INTERVAL notation.

19. $f(x) = x^2 - 5$

20. $f(x) = -\sqrt{x+3}$

21. $f(x) = 3 \sin x$

22. $f(x) = \frac{2}{x-1}$

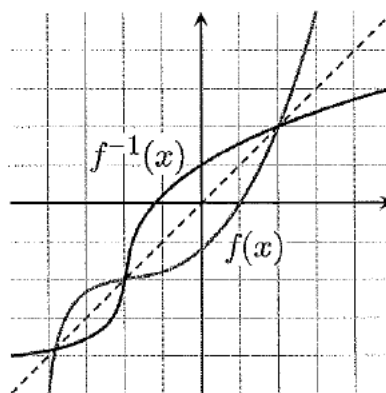
Assignment 5: Inverses

INVERSES

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value. Recall $f^{-1}(x)$ is defined as the inverse of $f(x)$

Example 1:

$f(x) = \sqrt[3]{x+1}$	Rewrite $f(x)$ as y
$y = \sqrt[3]{x+1}$	Switch x and y
$x = \sqrt[3]{y+1}$	Solve for your new y
$(x)^3 = (\sqrt[3]{y+1})^3$	Cube both sides
$x^3 = y+1$	Simplify
$y = x^3 - 1$	Solve for y
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation



Find the inverse for each function.

23. $f(x) = 2x + 1$

24. $f(x) = \frac{x^2}{3}$

25. $g(x) = \frac{5}{x-2}$

26. $y = \sqrt{4-x} + 1$

27. If the graph of $f(x)$ has the point $(2, 7)$ then what is one point that will be on the graph of $f^{-1}(x)$?

28. Explain how the graphs of $f(x)$ and $f^{-1}(x)$ compare.

Assignment 6: Equation of a Line

EQUATION OF A LINE

Slope intercept form: $y = mx + b$	Vertical line: $x = c$ (slope is undefined)
Point-slope form: $y - y_1 = m(x - x_1)$	Horizontal line: $y = c$ (slope is 0)
* LEARN! We will use this formula frequently!	
Example: Write a linear equation that has a slope of $\frac{1}{2}$ and passes through the point (2, -6)	
Slope intercept form	Point-slope form
$y = \frac{1}{2}x + b$ Plug in $\frac{1}{2}$ for m	$y + 6 = \frac{1}{2}(x - 2)$ Plug in all variables
$-6 = \frac{1}{2}(2) + b$ Plug in the given ordered	$y = \frac{1}{2}x - 7$ Solve for y
$b = -7$ Solve for b	
$y = \frac{1}{2}x - 7$	

29. Determine the equation of a line passing through the point (5, -3) with an undefined slope.
30. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.
31. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of $\frac{2}{3}$.
32. Use point-slope form to find a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.
33. Use point-slope form to find a line perpendicular to $y = -2x + 9$ passing through the point (4, 7).
34. Find the equation of a line passing through the points (-3, 6) and (1, 2).
35. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3)

Assignment 8: Trigonometric Equations & Transformation of Functions

TRIGONOMETRIC EQUATIONS

Solve each of the equations for $0 \leq x < 2\pi$.

37. $\sin x = -\frac{1}{2}$

38. $2 \cos x = \sqrt{3}$

39. $4 \sin^2 x = 3$

**Recall $\sin^2 x = (\sin x)^2$

***Recall if $x^2 = 25$ then $x = \pm 5$

40. $2 \cos^2 x - 1 - \cos x = 0$ *Factor

TRANSFORMATION OF FUNCTIONS

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x - c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x + c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

41. Given $f(x) = x^2$ and $g(x) = (x - 3)^2 + 1$. How the does the graph of $g(x)$ differ from $f(x)$?

42. Write an equation for the function that has the shape of $f(x) = x^3$ but moved six units to the left and reflected over the x-axis.

43. If the ordered pair $(2, 4)$ is on the graph of $f(x)$, find one ordered pair that will be on the following functions:

a) $f(x) - 3$

b) $f(x - 3)$

c) $2f(x)$

d) $f(x - 2) + 1$

e) $-f(x)$

Assignment 9: Vertical Asymptotes

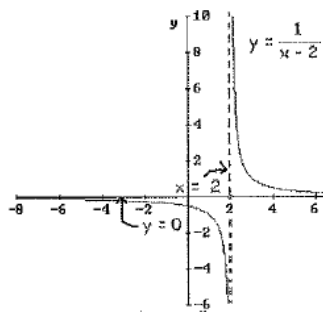
VERTICAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).

Write a vertical asymptotes as a line in the form $x =$

Example: Find the vertical asymptote of $y = \frac{1}{x-2}$

Since when $x = 2$ the function is in the form $1/0$ then the vertical line $x = 2$ is a vertical asymptote of the function.



44. $f(x) = \frac{1}{x^2}$

45. $f(x) = \frac{x^2}{x^2 - 4}$

46. $f(x) = \frac{2+x}{x^2(1-x)}$

47. $f(x) = \frac{4-x}{x^2 - 16}$

48. $f(x) = \frac{x-1}{x^2 + x - 2}$

49. $f(x) = \frac{5x+20}{x^2 - 16}$

Assignment 10: Horizontal Asymptotes

HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Example: $y = \frac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Example: $y = \frac{2x^2 + x - 1}{3x^2 + 4}$ (As x becomes very large or very negative the value of this function will approach $2/3$). Thus there is a horizontal asymptote at $y = \frac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \frac{2x^2 + x - 1}{3x - 3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Determine all Horizontal Asymptotes.

50. $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

51. $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

52. $f(x) = \frac{4x^2}{3x^2 - 7}$

53. $f(x) = \frac{(2x-5)^2}{x^2-x}$

54. $f(x) = \frac{-3x+1}{\sqrt{x^2+x}}$ * Remember $\sqrt{x^2} = \pm x$

This is very important in the use of limits.

EXPONENTIAL FUNCTIONS

Example: Solve for x

$$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$$

$$(2^2)^{x+1} = (2^{-1})^{3x-2} \quad \text{Get a common base}$$

$$2^{2x+2} = 2^{-3x+2} \quad \text{Simplify}$$

$$2x+2 = -3x+2 \quad \text{Set exponents equal}$$

$$x = 0 \quad \text{Solve for x}$$

Solve for x:

55. $3^{3x+5} = 9^{2x+1}$

56. $\left(\frac{1}{9}\right)^x = 27^{2x+4}$

57. $\left(\frac{1}{6}\right)^x = 216$

LOGARITHMS

The statement $y = b^x$ can be written as $x = \log_b y$. They mean the same thing.

REMEMBER: A LOGARITHM IS AN EXPONENT

Recall $\ln x = \log_e x$

The value of e is 2.718281828... or $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$

Evaluate the following logarithms

58. $\log_7 7$

59. $\log_3 27$

60. $\log_2 \frac{1}{32}$

61. $\log_{25} 5$

62. $\log_9 1$

63. $\log_4 8$

64. $\ln \sqrt{e}$

65. $\ln \frac{1}{e}$

Example: Evaluate the following logarithms

$\log_2 8 = ?$

In exponential for this is $2^? = 8$

Therefore $? = 3$

Thus $\log_2 8 = 3$

Assignment 12: Properties of Logarithms

PROPERTIES OF LOGARITHMS

$\log_b xy = \log_b x + \log_b y$	$\log_b \frac{x}{y} = \log_b x - \log_b y$	$\log_b x^y = y \log_b x$	$b^{\log_b x} = x$
Examples:			
Expand $\log_4 16x$	Condense $\ln y - 2 \ln R$	Expand $\log_2 7x^5$	
$\log_4 16 + \log_4 x$	$\ln y - \ln R^2$	$\log_2 7 + \log_2 x^5$	
$2 + \log_4 x$	$\ln \frac{y}{R^2}$	$\log_2 7 + 5 \log_2 x$	

Use the properties of logarithms to evaluate the following

66. $\log_2 2^5$

67. $\ln e^3$

68. $\log_2 8^3$

69. $\log_3 \sqrt[4]{9}$

70. $2^{\log_2 10}$

71. $e^{\ln 8}$

72. $9 \ln e^2$

73. $\log_9 9^3$

74. $\log_{10} 25 + \log_{10} 4$

75. $\log_2 40 - \log_2 5$

76. $\log_2 (\sqrt{2})^5$

Assignment 13: Even & Odd Functions

EVEN AND ODD FUNCTIONS

Recall:

Even functions are functions that are symmetric over the y-axis.

To determine algebraically we find out if $f(x) = f(-x)$

(*Think about it what happens to the coordinate $(x, f(x))$ when reflected across the y-axis*)

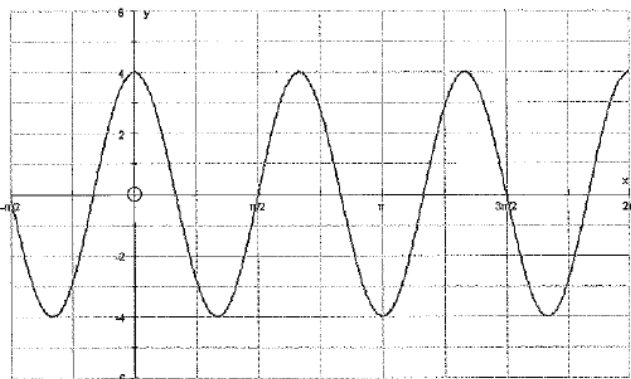
Odd functions are functions that are symmetric about the origin.

To determine algebraically we find out if $f(-x) = -f(x)$

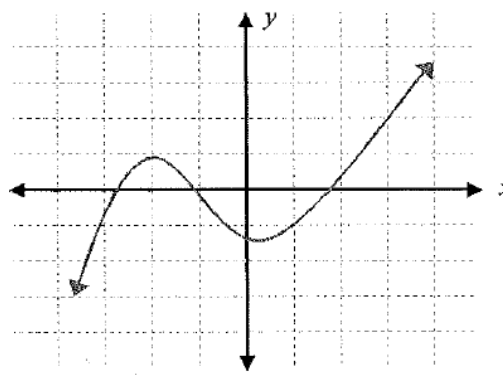
(*Think about it what happens to the coordinate $(x, f(x))$ when reflected over the origin*)

State whether the following graphs are even, odd or neither, show ALL work.

77. _____



78. _____



79. _____
 $f(x) = 2x^4 - 5x^2$

80. _____
 $g(x) = x^5 - 3x^3 + x$

81. _____
 $h(x) = 2x^2 - 5x + 3$

82. _____
 $j(x) = 2 \cos x$

83. _____
 $k(x) = \sin x + 4$

84. _____
 $l(x) = \cos x - 3$

Assignment 14: Fill in the Unit Circle

Fill in The Unit Circle

